Computing the Resolution of a Gamma Camera

D. Cecchin[a], S. De Marchi[b], **D. Poggiali**[c], L. Riccardi [d], et al.

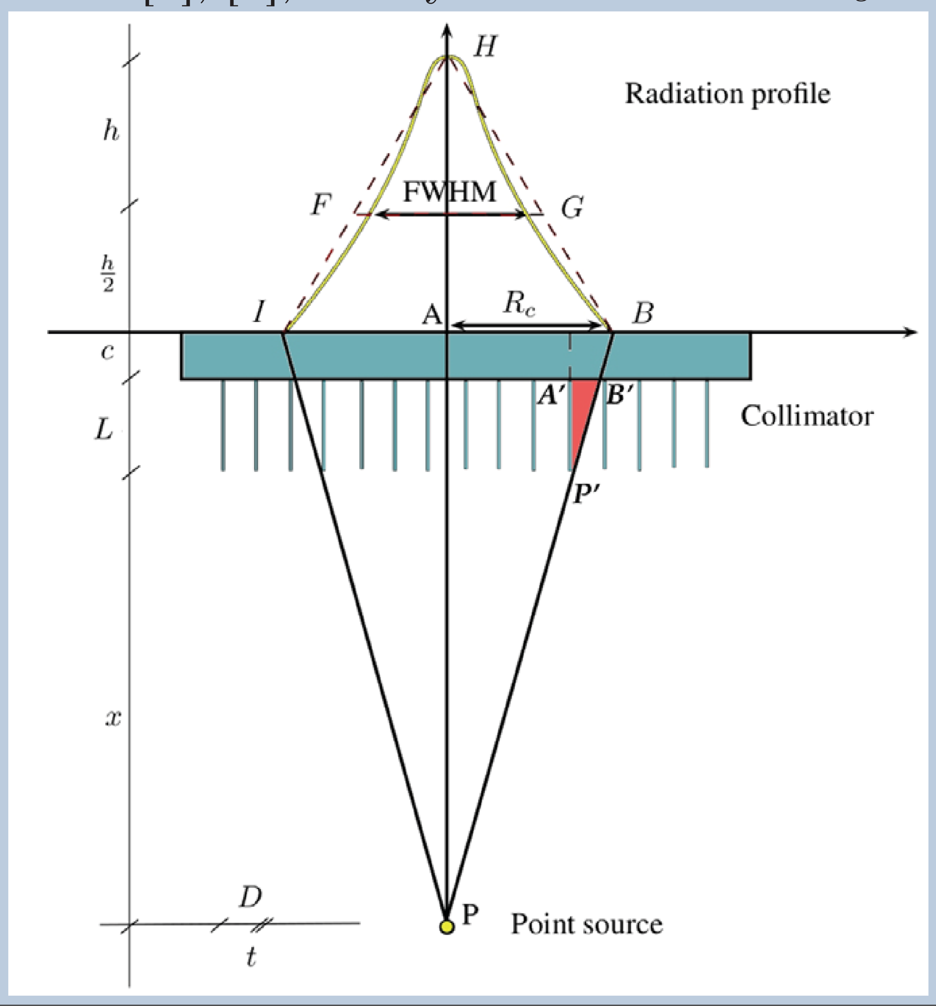
[a]Department of Medicine, University of Padua (Italy). diego.cecchin@unipd.it
[b]Department of Mathematics, University of Padua (Italy). demarchi@math.unipd.it
[c]Department of Mathematics, University of Padua (Italy). poggiali.davide@gmail.com
[d]Medical Physics Unit, Veneto Institute of Oncology IOV IRCSS (Italy). lucia.riccardi@unipd.it

Abstract

It is well known that resolution varies as a function of distance and gamma camera's characteristics. Frequently, however, **manufacturers provides only few pre-calculated values of resolution** and typically obtained in non-clinical like situations. From a diagnostic point of view it is useful to know which is the expected resolution of a gamma camera to decide whether it is worth scanning that patient with a "little lesion" or not.
A reliable way to calculate the theoretical resolution of a gamma camera at different distances (***Analytical resolution***) and compare it to real-data-obtained FWHM (***Experimental resolution***) is presented.

Analytical resolution

As in [1], [6], the System resolution R_s depends on the *collimator resolution* R_c and on the *intrinsic resolution* R_i.



Using the convolution theory, see [1], we get

$$R_s = \sqrt{R_c^2 + R_i^2}\,. \tag{1}$$

The similitude of triangles $P\overset{\triangle}{A}B$ and $P'\overset{\triangle}{A'}B$ gives

$$R_c = D\left(1 + \frac{x+c}{L_{eff}}\right). \tag{2}$$

The parameters D, c, L_{eff}, R_i are provided by the producers of the gamma camera.

Three Methods for Experimental resolution

Data obtain from a static scintigraphy of a line source is a $N \times N$ matrix. The user selcts a $N \times J$ submatrix, where the width of the line seems constant. For each j-th row of the chosen submatrix data, $FWHM_j$ was calculated from data $(x_i, y_i)_{i=1,\dots,N}$ using one of the three methods described below. The $FWHM$ value was assessed as average of $FWHM_j$ $(j = 1, \dots, J)$. As estimation of the absolute and relative errors the standard deviation σ and the *variation coefficient vc* were respectively calculated.For each method a quadratic *cost* has been defined case by case, to quantify the accuracy.

1. Direct calculation: The maximum pixel value $h = \max(y_i)$ and the relative argument $\tilde{x}$ were found. Two points $z_1 < \tilde{x}$ and $z_2 > \tilde{x}$, which are the closest to $\frac{h}{2}$, were used and their distance $FWHM_j = |z_1 - z_2|$. were determined. For this case the following cost was defined:

$$C_1(z_1, z_2) = \frac{(y(z_1) - h/2)^2 + (y(z_2) - h/2)^2}{2}\,. \tag{3}$$

2. Global interpolation - Gaussian: Data (x_i, y_i) are modeled as a deterministic function with a small level of noise. The *nonlinear least-squares* approach is used, see [5]:

$$\bar{a}^* = \arg\min_{\bar{a}\in\mathbb{R}^n} J(\bar{a}) = \arg\min_{\bar{a}\in\mathbb{R}^n} \|y_i - f_{\bar{a}}(x_i)\|^2\,.$$

The cost used in this method is

$$C_2(\bar{a}) = \frac{J(\bar{a}^*)}{N}\,. \tag{4}$$

The *gaussian* function was used as in [7]: $f_{\bar{a}}(x) = a_1 e^{-a_2^2 x^2}$ which has resolution $FWHM_j = 2\frac{\sqrt{\log(2)}}{|a_2|}$.

3. Local interpolation - Splines: Cubic splines $s(x)$ were chosen for their well-known approximation properties, see [3], [4].
As in method 1 the algorithm searches two points z_1 and z_2 whose distance from the half of the maximum is minimal:

$$z_i = \arg\min_{x\in I_i} |s(x) - h/2|\,, \qquad i = 1, 2$$

where I_1 and I_2 are sets of $5\cdot 10^4$ equispaced points of the intervals $(x_1, \tilde{x})$ and $(\tilde{x}, x_N)$ respectively. The distance of these two points gives a good estimate of the $FWHM_j = |z_1 - z_2|$. The cost is defined as in (3).

Results of the three methods

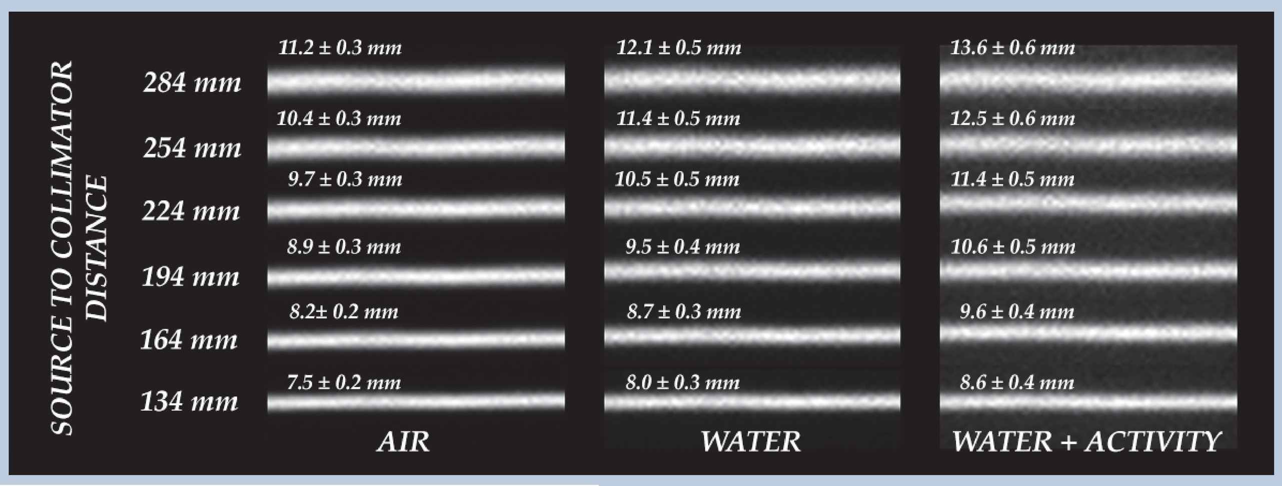
The direct method, although very simple and fast, demonstrates an elevated cost as compared to the others. Results obtained using local and global interpolation are nearly identical in terms of FWHM, σ and vc although different dealing with mean cost. This highlights the higher reliability of local interpolation.

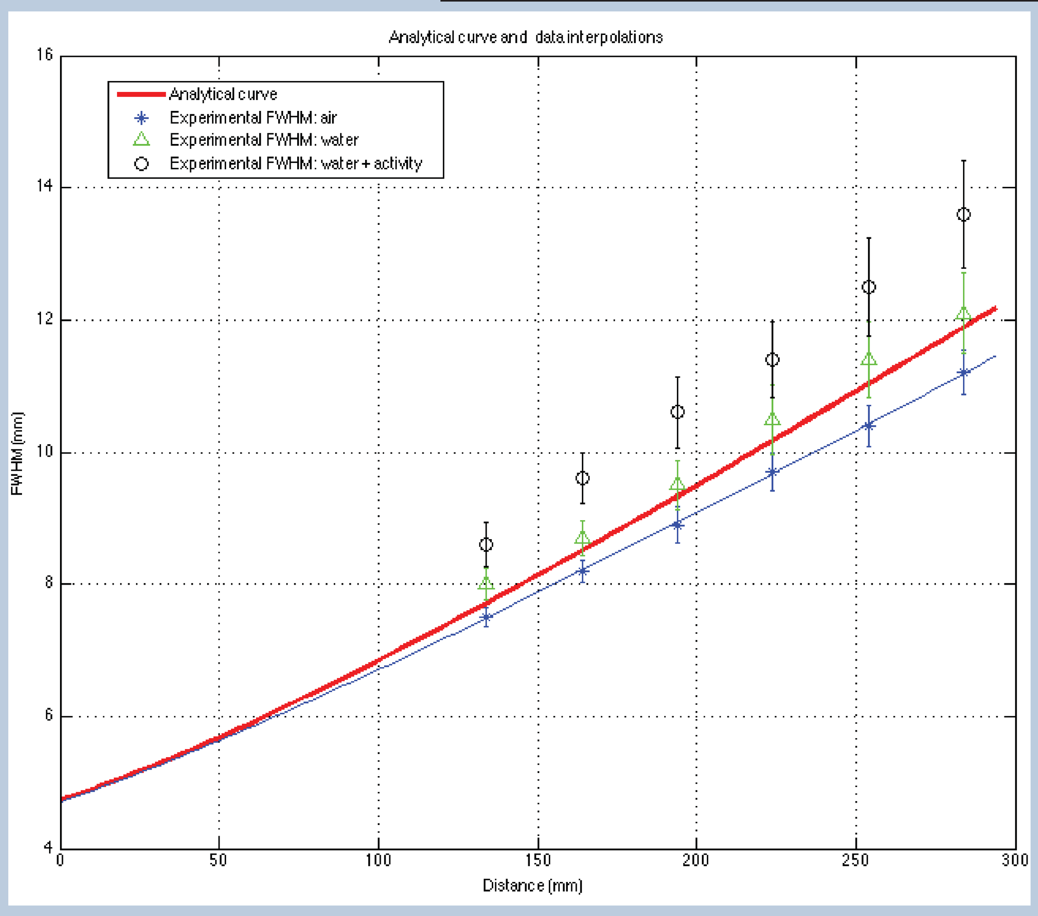
	Experimental FWHM						
	radius (mm)	134	164	194	224	254	284
Direct Met.	fwhm (mm)	7.31	8.13	8.96	9.71	10.62	11.37
	σ (mm)	0.80	1.19	0.87	0.87	1.18	0.80
	vc (%)	10.9	14.6	9.7	9.0	11.1	7.0
	mean cost	147543	137062	88423	66376	44393	34223
Global Int.	fwhm (mm)	7.53	8.20	8.88	9.65	10.54	11.35
	σ (mm)	0.24	0.22	0.22	0.21	0.33	0.30
	vc (%)	3.2	2.7	2.5	2.2	3.1	2.7
	mean cost	3.75	3.11	2.33	2.06	2.15	1.99
Local Int.	fwhm (mm)	7.50	8.23	8.92	9.70	10.54	11.36
	σ (mm)	0.26	0.27	0.26	0.26	0.37	0.31
	vc (%)	3.4	3.3	2.9	2.6	3.5	2.7
	mean cost	1.55	0.95	0.79	0.49	0.35	0.25
	Analytical FWHM						
	fwhm (mm)	7.70	8.50	9.33	10.17	11.03	11.89

Results with different scatter conditions

The local interpolation method was used to calculate the FWHM in three different scatter conditions:

- **in air**
- **in water**
- **in water + activity**





The aim is to check the reliability of the Analytical formula in clinical-like conditions.
Analytical formula offers quite accurate values of resolution in case of low scatter. On the other hand when dealing with high scatter conditions, it gives a significant underestimation of the real FHMM.

The software package

An open source Scilab package, `Resolution Calculator 0.1Beta`, has been implemented and tested and is freely available at: http://goo.gl/siWVbg or following this QR.

References

[1] Cherry S. M., Sorenson J., Phelps M., *Physics in Nuclear Medicine*, Saunders, 2012, Ch.15, App.G

[2] Cecchin D., Poggiali D., Riccardi L., Turco P., Bui F. and De Marchi S., *Analytical and experimental resolution of a gamma camera: theoretical and practical issues*, submitted, 2013

[3] de Boor C., *A Practical Guide to Splines*, Springer, 2001, Ch.4

[4] Lancaster P., Šalkauskas K., *Curve and Surface Fitting: An Introduction*, Elsevier, 1986, Ch.3

[5] Nocedal J., Wright S. *Numerical Optimization 2nd ed.*, Springer, 2006, Ch.10.3

[6] Soreson J.A., Phelps M.E., *Physics in nuclear medicine*, Second edition, Grune & Stratton, 1987, Ch.3

[7] Zaidi H., *Quantitative Analysis in Nuclear Medicine Imaging*, Springer, 2006, Ch.5